Remote synchronization: detailed account of a peculiar pattern-formation mechanism

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Context

A “rewarding” experiment about relationship(s) between structural connectivity and synchronization in an electronic network
What is remote synchronization?

Synchronised

Non-synchronised

A  B  C

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Remote synchronization from mismatches

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Remote synchronization in star networks

A. Bergner, M. Frasca, G. Scitto, A. Buscarino, E. J. Ngamga, L. Fortuna, and J. Kurths

\[ \dot{u}_i = (\alpha + i \omega_i - |u_i|^2)u_i + \frac{\sigma}{d_i} \sum_{j=1}^{N} a_{ij}(u_j - u_i) \]

Image credit: cited study, © APS
Remote synchronization as morphogenesis
Remote synchronization in brain networks?

Sensory-motor network: directly wired

Default-mode network: emergent

No direct anatomical link to posterior areas. Remotely synchronized?

Image source: Rosazza & Minati, 2011; © Springer Verlag
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A simple, reconfigurable non-linear network

![Diagram of a simple, reconfigurable non-linear network](image)
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Oscillator equations

\[
\begin{align*}
\frac{dv_1}{dt} &= \Gamma \left( 2\pi F_1 (G_4 v_4 + G_5 v_5 - v_1), v_1 \right) \\
\frac{dv_2}{dt} &= \Gamma \left( 2\pi F_2 (G_1 v_6 - v_2), v_2 \right) \\
\frac{dv_3}{dt} &= \Gamma \left( K_1 v_6, v_3 \right) \\
\frac{dv_4}{dt} &= \Gamma \left( 2\pi F_3 (G_2 v_2 + G_3 v_3 - v_4), v_4 \right) \\
\frac{dv_5}{dt} &= \Gamma \left( K_2 v_2, v_5 \right) \\
\frac{dv_6}{dt} &= \Gamma \left( 2\pi F_3 (G_4 v_1 + G_5 v_4 + G_6 v_6 - v_6), v_6 \right)
\end{align*}
\]

\[
\Gamma (x, y) = R (x) H (V_2 - y) - R (-x) H (V_2 + y)
\]

Parametric mismatch

\sim 0.5\% in physical system
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Applications in versatile pattern generation

Local Pattern Generators (LPGs)
Control parameters: $P_3$, $P_4$ and $P_5$.

Central Pattern Generator (CPG)
Control parameters: $P_1$ and $P_2$.

Local Pattern Generators (LPGs)
Control parameters: $P_3$, $P_4$ and $P_5$.

Image source: Minati et al., IEEE Access 2018. © IEEE
Phase vs. amplitude synchronization

Phase coherence  \( r_{ij} = |\langle e^{i[\varphi_i(t) - \varphi_j(t)]} \rangle| \)

Instantaneous amplitude (envelope)
\[
v_i(t) + i \hat{v}_i(t) = A_i(t)e^{i\varphi_i(t)}
\]
where \( \hat{v}_i \) is the Hilbert transform of \( v_i(t) \)
\[
\hat{v}_i(t) = \frac{1}{\pi} \text{p.v.} \left[ \int_{-\infty}^{\infty} \frac{v_i(\tau)}{t-\tau} d\tau \right]
\]
and where p.v. denotes the Cauchy principal value of the integral\(^{18}\).

Maximum cross-correlation or mutual information
\[
C_{XY}(\tau) = \frac{k_{XY}(\tau)}{\sqrt{\sigma_X^2 \sigma_Y^2}} \quad N_{XY}(d) = \frac{I_{XY}(d)}{\sqrt{H_X H_Y}}
\]
Numerical simulations reveal three regimes

- $G_6 = 0.196$
- $G_7 = -1.365$

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Image source: Minati et al., CHAOS 2015. © AIP
Numerical simulations reveal three regimes

a: $G_6 = 0.196$, $G_7 = -1.365$

b: $G_6 = 0.096$, $G_7 = -1.53$

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Numerical simulations reveal three regimes

a: $G_6 = 0.196$, $G_7 = -1.365$

b: $G_6 = 0.096$, $G_7 = -1.53$

c: $G_6 = 0.188$, $G_7 = 1.14$

Broadband chaos

Quasi-periodicity

Narrowband chaos

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Numerical simulations reveal three regimes

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Numerical simulations reveal three regimes

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Image source: Minati et al., CHAOS 2018. © AIP
Effect of parametric mismatches
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Experimental implementation

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Experimental implementation

Top-level FPAA architecture
Experimental implementation

The Configurable Analog Module (CAM)
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Experimental implementation

- **GainHalf**
  - Half-cycle
- **GainHold**
  - Inverting only
- **GainInv**
  - Continuous Time
- **SumInv**
  - Up to three inputs
- **SumDiff (SumHalf)**
  - Up to four inputs
  - Add or subtract since input branches can be inverting or non-inverting
- **RectifierFilter**
  - Full Wave/Half Wave
  - Inverting/non-inverting
- **RectifierHalf**
  - Full Wave/Half Wave
  - Inverting/non-inverting
- **RectifierHold**
  - Half Wave Inverting only

Upon permission and courtesy of Anadigm, Inc.
Experimental implementation

- **FilterBilinear – One pole**
  - Low Pass/High Pass/All Pass
- **FilterBiquad – Two poles**
  - Low Pass/High Pass/Band Pass/Band Stop
  - Automatically chooses from multiple circuit topologies
- **Differentiator**
  - Output voltage slews – see documentation
- **Integrator**
  - Optional reset
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Experimental implementation

- **Multiplier**
  - Uses SAR (Input Y is quantized)
  - Subject to internal reference voltage error
  - Optional sample and hold on input X to equalize sampling time of two inputs (uses chip resources)

- **Comparator**
  - Single/Dual Input
  - Variable Reference

- **Hold – Sample and hold**

- **Oscillator Sine**
  - Subject to internal reference voltage error

- **Voltage (+/- 3 VDC)**
  - Subject to internal reference voltage error
Experimental implementation

Continuous-value, discrete-time

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Experimental implementation

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From: A. Buscarino et al., A Concise Guide to Chaotic Electronic Circuits, 73, SpringerBriefs in Applied Sciences and Technology
Experimental implementation

A soft of “Chimera”: an analog plug-in system for digital computer

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Remote sync. close to quasi-periodicity

Correlation dimension ($D_2$)

Remote synchronization ($\eta$)

\[
\eta[r_{nm}] = \frac{\sum_{m=1}^{32} \sum_{n=1}^{32} \Theta[H(r-r')]_{nm}}{\sum_{m=1}^{32} \sum_{n=1}^{32} H(r_{nm} - r')},
\]

Image source: Minati et al., CHAOS 2015. © AIP
Experimental data - basics

Spectrogram

Phase sync.

Amplitude sync.

Reminiscent of a diffraction pattern?

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Non-stationarity

Adjacent time-windows...
Non-stationarity

...reminiscent of observations in resting-state functional MRI

Experimental data – focus on spectrum

Two concomitant spectral relations:

1) \( f_B = f_H - f_C = f_C - f_L \)

2) \( f_H = f_L + f_C \) \( \rightarrow f_L = f_B \)

Reminiscent of classic AM modulation!

Lower sideband and baseband overlap!
From synchronization to causality

- Regression of the present of the target on its own past:
  \[ e_{j|i,n} = y_{j,n} - E[y_{j,n} \mid y_{j,n}] \quad \text{\( \Rightarrow \)} \quad \lambda_{j|i} = E[e_{j|i,n}^2] \]

- Regression of the present of the target on its past and the past of the source:
  \[ e_{j|ji,n} = y_{j,n} - E[y_{j,n} \mid y_{j,n}, y_{i,n}] \quad \text{\( \Rightarrow \)} \quad \lambda_{j|ji} = E[e_{j|ji,n}^2] \]

Granger causality (GC)

\[ F_{i\rightarrow j} = \ln \frac{\lambda_{j|i}}{\lambda_{j|ji}} \]

Transfer Entropy (TE)

\[ T_{i\rightarrow j} = \frac{1}{2} \ln \frac{\lambda_{j|i}}{\lambda_{j|ji}} \]


Mutual information and causality

Mutual info.

Granger with quadratic+ cross-terms

Linear Granger

Transfer Entropy

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Effect of “lesioning” by noise injection

- Voltage $v_j(t)$ / V vs. Time $t$/ms
- $\langle \Delta \max [C_{ij}(\tau)]_{\tau \geq 0} \rangle$
- $\Delta \max [C_{ij}(\tau)]_{\tau \geq 0}$ for Node $\Delta i$
- $\Delta \max [C_{ij}(\tau)]_{\tau \geq 0}$ for Node $\Delta j$

Image source: Minati et al., CHAOS 2018. © AIP
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Ring size and auxiliary system simulations

![Graphs showing ring size and auxiliary system simulations](Image source: Minati et al., CHAOS 2018. © AIP)
Propagation of external perturbations

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Image source: Minati et al., CHAOS 2018. © AIP, ANT Neuro
Simplified chain model

1) An open network is considered in the form of a chain.
2) Two dynamical equations are removed.
3) $\Gamma(x, y)$ is removed for all voltages except $v_3$.
4) The parameters are set identically across all nodes.
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Simplified chain model

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Demodulation and interference
Demodulation and interference

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Image source: Minati et al., CHAOS 2018. © AIP
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Instancing filters at specific points of chain

![Graphs showing instancing filters at specific points of chain](Image source: Minati et al., CHAOS 2018. © AIP)
Revised Granger model: baseband + sideband

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Image source: Minati et al., CHAOS 2018. © AIP
Small-world features, nonetheless...

$$S^{WS} = \gamma^{WS}_g / \lambda_g = \left( C^{WS}_gL_{rand} \right) / \left( C^{WS}_{rand}L_g \right)$$

a)  

b)  

C)
Small-world features, nonetheless...

Small-worldness in the brain (and not only) is an efficient trade-off!

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Conclusions

1) A complex mechanism of pattern generation was demonstrated.

2) Is this just “apparent” remoteness? Central importance of measure choice…

3) To what systems may such mechanism apply? Broadband vs. narrowband chaos, spectral relationships
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Remote synchronization in star networks

\[ \dot{u}_i = (\alpha + i\omega_i - |u_i|^2)u_i + \frac{\sigma}{d_i} \sum_{j=1}^{N} a_{ij}(u_j - u_i) \]

Image credit: cited study, © APS
Non-monotonic effect of the coupling strength

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Fading of remote synchronization in tree networks of Stuart-Landau oscillators

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\[
\begin{align*}
\dot{x}_i &= \alpha x_i - \omega_i y_i - x_i(x_i^2 + y_i^2) + \frac{\sigma}{k_i} \sum_{j=1}^{N} a_{ij} (x_j - x_i), \\
\dot{y}_i &= \omega_i x_i + \alpha y_i - y_i(x_i^2 + y_i^2) + \frac{\sigma}{k_i} \sum_{j=1}^{N} a_{ij} (y_j - y_i),
\end{align*}
\]

Image credit: Karakaya et al. PRE 2019, © APS
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Non-monotonic effect of the coupling strength

$$\dot{x}_i = \alpha x_i - \omega y_i - x_i(x_i^2 + y_i^2) + \frac{\sigma}{k_i} \sum_{j=1}^{N} a_{ij} (x_j - x_i),$$

$$\dot{y}_i = \omega x_i + \alpha y_i - y_i(x_i^2 + y_i^2) + \frac{\sigma}{k_i} \sum_{j=1}^{N} a_{ij} (y_j - y_i),$$
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Non-monotonic effect of the coupling strength

\[
\begin{align*}
    r_{\text{indirect}} &= \frac{2}{(N-1)(N-2)} \sum_{i=2, j>i}^{N} r_{ij}, \\
    r_{\text{direct}} &= \frac{1}{(N-1)} \sum_{j=2}^{N} r_{ij}.
\end{align*}
\]
Non-monotonic effect of the coupling strength

Image credit: Karakaya et al. PRE 2019, © APS
Thank you for your attention

References:

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