UNIVERSITY OF TRENTO - Italy
Department of Economics and Management

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GOFCP 2019

4th WORKSHOP ON GOODNESS-OF-FIT, CHANGE-POINT AND RELATED PROBLEMS

Trento, 6-8 September 2019

BOOK OF ABSTRACTS

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GOFCP 2019

Preface

The 4th Workshop on *Goodness-of-fit, Change Point and related problems* is held in Trento, at the Department of Economics and Management of the University of Trento. The workshop follows those of Sevilla (2012), Athens (2015) and Bad Herrenalb (2017).

This edition counts 35 speakers and 9 poster presentations of younger researchers. The variety of topics and the quality of the works presented show the capacity of the themes of the workshop to keep up-to-date with the modern developments of statistical theory and applications as well as to attract talented researchers of all ages.

The workshop will be also the occasion for a formal tribute to Marie Hušková and Winfried Stute in recognition of their career constellated with important contributions to statistical theory.

I would like to remind the fundamental role played by all members of the Scientific Program Committee (SPC) and Simos Meintanis in designing the final program of the workshop. Local details and logistics of this event were carried out by the Local Organizing Committee (LOC). Special thanks goes to Flavio Santi for all his technical support. The staff at DEM and the Communications and Events office of University of Trento provided all support with administrative issues and the web page.

Last but not least, thanks are due to our sponsors: Department of Economics and Management, University of Trento and Department of Statistics, North West University.

Emanuele Taufer
Chair of SPC & LOC

PRESENTED PAPERS

Abstracts of Presented Papers[1]

[1]Papers are listed in alphabetical order by authors. For each paper, the name of the speaker is typed in bold font.

A Kolmogorov-Smirnov approach to goodness-of-fit to contamination models

Eustasio del Barrio,[1] *IMUVa, University of Valladolid*
Hristo Inouzhe, *IMUVa, University of Valladolid*
Carlos Matrán, *IMUVa, University of Valladolid*

Abstract. Often in statistical practice a model offers a good description of the data even though it is not the "true" random generator. A classical test of fit would reject such a model if the sample is large enough. We consider a more flexible approach based on contamination neighbourhoods around a model. Using trimming methods and the Kolmogorov metric we introduce tests of fit to some contamination neighbourhoods which have uniformly exponentially small type I and type II error probabilities. We prove that, under suitable regularity conditions, the normalized difference between our estimator and the estimated value converges weakly to the supremum of a Gaussian process. As an application we explore methods of comparison between descriptive models based on the paradigm of model falseness.

Keywords: Goodness-of-fit, contamination neighbourhood, trimming methods, uniformly consistent test

1 CONTAMINATION MODELS, TRIMMING METHODS AND MODEL VALIDATION

Model validation has been a main goal of statistical analysis since early times. It has also been subject to important criticism for a long time. Classical goodness of fit tests try to establish if there is enough statistical evidence to reject the null hypothesis. These procedures behave fairly well for moderate data sizes, but can become excessively rigid in the presence of large sample sizes. This was already noted by Berkson for the χ^2 statistic in 1938. The issue has been approached by different authors, coming to the conclusion that we should broaden the null hypothesis to include useful models, even if they are not exactly the true model. Often, this is also accompanied by a gain in robustness. But even under the paradigm of model falseness, rejecting a model would be not a satisfactory goal. If all models are false, and at a certain point, with enough data, we will be able to reject the model at hand,

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we could provide some measure of how useful that model is or of how good it is compared to other models.

A notion of model usefulness can be based on Huber's contamination neighbourhoods, namely,

$$\mathcal{V}_\alpha(P_0) = \{(1-\alpha)P_0 + \alpha Q : Q \in \mathcal{P}\}.$$

Contamination neighbourhoods are related to trimmings by $P \in \mathcal{V}_\alpha(P_0)$ $\Leftrightarrow P_0 \in R_\alpha(P)$, the set of α-trimmings of the probability P,

$$R_\alpha(P) = \{Q \in \mathcal{P} : Q \ll P,\ \tfrac{dQ}{dP} \leq \tfrac{1}{1-\alpha}\ P\text{-a.s.}\}.$$

We will consider that P_0 is a useful model for P if P belongs to or is close to $\mathcal{V}_\alpha(P_0)$. The duality between contamination neighbourhoods and trimmings allows to take advantage of the good mathematical properties of the latter.

In this work we pursue two main goals. The first one is to develop a robust hypothesis testing procedure based on the previous considerations using the Kolmogorov (or L_∞-)distance between d.f.'s on the line, namely,

$$d_K(F,G) = \sup_{x\in\mathbb{R}} |F(x) - G(x)|.$$

The second is to use this procedure under the paradigm of a false-model world to derive some useful tools for comparing models.

For the first goal we consider testing problems of the form

$$H_0 : d_K(F_0, R_\alpha(F)) \geq \rho_1 \quad \text{vs.} \quad H_1 : d_K(F_0, R_\alpha(F)) < \rho_2, \tag{1.1}$$

with $0 \leq \rho_2 < \rho_1$. We introduce a uniformly consistent test (type I and type II error probabilities tend to 0 uniformly) for which exact exponential bounds for the error probabilities are available. Additionally, we provide asymptotic theory for $d_K(F_0, R_\alpha(F_n))$ that allows to compute confidence bounds for $d_K(F_0, R_\alpha(F))$. This asymptotic theory can also be used to deal with the second goal in terms of so-called *credibility indices*. Here we show how to use our asymptotic theory for $d_K(F_0, R_\alpha(F_n))$ to obtain further information about the credibility indices.

This work is based on del Barrio, Inouzhe and Matrán (2019).

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Asymptotic Variance of Test Statistics in the ML and QML Frameworks

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Abstract. In this study, we consider the test statistics that can be written as the sample average of data and derive their limiting distribution under the maximum likelihood (ML) and the quasimaximum likelihood (QML) frameworks. We first generalize the asymptotic variance formula suggested in Pierce (1982) in the ML framework and illustrate its applications through some wellknown test statistics: (i) the skewness statistic, (ii) the kurtosis statistic, (iii) the Cox statistic, (iv) the information matrix test statistic, and (v) the Durbin's h-statistic. We next provide a similar result in the QML setting and illustrate its applications by providing two examples. Illustrations show the simplicity and the effectiveness of our results for the asymptotic variance of test statistics, and therefore, they are recommended for practical applications.

Keywords: Variance, Asymptotic variance, MLE, QMLE, Inference, Test statistics, Skewness statistic, Kurtosis statistic, The Cox statistic, The information matrix test, Durbin's h-statistic

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On ridge estimation with dependent noise

Jan Beran,[1] *Department of Mathematics and Statistics, University of Konstanz*
Klaus Telkmann, *Department of Statistics, University of California, Irvine*

Abstract. Ridge detection is a standard task in image processing. Early developments of the area can be found mainly in the engineering literature. In recent years, with the emergence of statistical methods for big data, ridge detection has become one of the focal points in statistical reasearch. In this talk we give a brief review of the area, with special emphasis on ridge estimation with dependent noise. In particular, under strong dependence, a reduction principle for the empirical process and for kernel estimators of multivariate derivatives of the marginal density function can be derived. Moreover, functional limit theorems for estimated eigenvalues and eigenvectors can be obtained. The asymptotic results lead to simultaneous confidence sets for density ridges.

Keywords: ridge; kernel density estimation; multivariate time series; long-range dependence; eigenvalues; functional limit theorem; reduction principle

1 INTRODUCTION

Image analysis and computer vision play an essential role in modern science and technology. One of the standard tasks is ridge detection. The topic has been developed mainly in the applied literature, see e.g. Haralick (1983), Eberly et al. (1994), Eberly (1996), Lindeberg (1998). In recent years, statistical literature on ridge detection has increased considerably, in particular in the context of topological statistics. For an early reference see Hall et al. (1992). Statistical inference for density ridges has been discussed mainly under i.i.d. assumptions, see e.g. Chen et al. (2015), Chen et al. (2015) and Qiao and Polonik (2016). In contrast, Telkmann (2018) and Beran and Telkmann (2019) consider situations where error processes are strongly dependent. For related results also see Beran and Telkmann (2018) and Beran and Schumm (2017); for recent literature on processes with strong (or long-range) dependence see e.g. Beran et al. (2013) and Giraitis et al. (2012). More specifically, let $\mu_1, \ldots, \mu_n$ be a sample generated by a probability distribution p_μ with compact support $M \subset \mathbb{R}^m$ and

$$\mathbf{Y}_t = (Y_{t,1}, ..., Y_{t,m})^T = \mu_t + \mathbf{X}_t \quad (t \in \mathbb{Z}),$$

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where $\mathbf{X}_t = (X_{t,1}, ..., X_{t,m})^T$ $(t \in \mathbb{Z})$ is a zero mean stationary error process and the marginal probability density function of $\mathbf{Y}_t$ is given by $p_Y = p_\mu \star p_X$. In typical applications, M is a $k-$dimensional manifold. For $k < m$ the $k-$dimensional ridge of p_Y is the set of points that are local maxima of p_Y in at least $m - k$ directions. As it turns out, strong dependence simplifies the construction of simultaneous confidence regions for density ridges. A multivariate reduction principle for empirical processes and for kernel estimators, together with functional limit theorems for estimated eigenvectors and eigenvalues of the Hessian matrix, provide the theoretical basis for the construction of simultaneous bounds.

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A new fixed point characterization of the Gamma distribution and associated goodness-of-fit tests

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Abstract. Applying the Stein-type distributional characterization of the Gamma distribution from the so-called density approach, we show the following characterization result [cf. Betsch and Ebner (2019)]: A positive random variable X with $\mathbb{E}\,|X| < \infty$ follows the Gamma law with shape parameter $k > 0$ and scale parameter $\lambda > 0$ if, and only if, the distribution function F of X has the form

$$F(t) = \mathbb{E}\left[\left(-\frac{k-1}{X} + \frac{1}{\lambda}\right)\min\{X, t\}\right], \quad t > 0.$$

Both quantities in this equation are easily estimated by empirical counterparts, and measuring the difference between those empirical versions in a suitable weighted L^2-distance immediately leads to a statistic which is used as a goodness-of-fit test. With appropriate asymptotic expansions for the estimators of the k- and λ-parameter, and by invoking the central limit theorem in Hilbert spaces, we derive the asymptotic null distribution. In doing so, we choose a setting that also allows to prove the asymptotic validity of the parametric bootstrap procedure that is implemented to run the tests. Furthermore, we establish the consistency of our tests in this bootstrap setting, and conduct a Monte Carlo simulation study to show the competitiveness to existing test procedures.

Note that since the density approach identity is not restricted to the Gamma distribution, we can extend the explicit formula above by showing that a very similar characterization holds for any distribution which is given through a continuously differentiable density function that satisfies some weak regularity conditions [consult Betsch and Ebner (2018)]. In the case of the normal distribution, our identity reduces to the characterization given by the zero-bias transformation which is widely used in the context of Stein's method. The corresponding goodness-of-fit tests for normality are studied in [Betsch and Ebner (2019)]. These tests are consistent against a large class of alternatives and show a strong performance in competitive simulations.

Keywords: Density Approach, Distributional Characterizations, Gamma Distribution, Goodness-of-fit Tests, Stein's Method

The poster corresponding to this abstract presents results from research that was carried out together with Bruno Ebner.

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Goodness of fit to the three parameter Weibull distribution

Alejandra Cabaña,[1] *Department of Mathematics, Autonomous University of Barcelona*
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Abstract. We present a family of goodness-of-fit tests for the three parameter Weibull family based on a quadratic form of the empirical process of standardised i.i.d sample. The tests can be focused against particular alternatives. Simulations and real data examples are included.

Keywords: G-o-F, Weibull family, estimated empirical process

We extend the metodology introduced in Cabaña and Cabaña (2003) where we developed consistent g-o-f tests to families of distributions that can be focused on privileged alternatives. The examples considered in the past were location/scale or purely scale families (such as the normal or exponential familes) and can be extended to more general families, with some regularity conditions,. As an example, we develop a test for the three-parameter Weibull distribution.

A random variable X has the Weibull distribution with parameters $a \in \mathbb{R}$ (location), $b \in \mathbb{R}^+$ (scale) and $c \in \mathbb{R}^+$ (shape) ($X \sim \mathrm{We}_{a,b,c}$) if its c.d.f. is

$$F_{a,b,c}(x) = 1 - \exp(-(\tfrac{x-a}{b})^c), x \geq a.$$

Given an i.i.d. sample $\{X_i\}_{i=1,2,\ldots,n} \sim F$, the fact that $\{X_i\}_{i=1,2,\ldots,n}$ is i.i.d $\mathrm{We}_{a,b,c}$ if and only if $\{\left(\frac{X_i-a}{b}\right)^c\}_{i=1,2,\ldots,n}$ is i.i.d. Standard Exponential can be used in order to construct tests for the null composite hypothesis F is Weibull. The usual procedure is to obtain estimations $\hat{a}, \hat{b}, \hat{c}$ of the parameters, and verify that the set of (now dependent) variables $\left\{\left(\frac{X_i-\hat{a}}{\hat{b}}\right)^{\hat{c}}\right\}_{i=1,2,\ldots,n}$ is approximately Standard Exponential.

The g-o-f to a family of distributions is greatly simplified when the densities depend smoothly on the parameters. This is not the case for the three-parameter Weibull, and thus, a different characterisation of the family is provided:

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The ordered set of random variables $(X_{(i)})_{i=0,1,\ldots,n}$ *is an ordered sample of* $We_{a,b,c}$ *if and only if the unordered set of variables*

$$\left\{Z_{(0)} := (n+1)\left(\frac{X_{(0)}-a}{b}\right)^c\right\} \cup \left\{Z_{(i)} := \left(\frac{X_{(i)}-a}{b}\right)^c - \left(\frac{X_{(0)}-a}{b}\right)^c\right\}_{i=1,2,\ldots,n}$$

is an i.i.d. sample of the Standard Exponential Distribution.

Under the null hypothesis, a suitable transformation of the data depending on the parameters should be distributed as a standard exponencial, and lie approximately in the tangente space when the transformation is based on estimations of the true unknown parameters. Departures from the null are reflected in distributions of the transformed data with components orthogonal to that space.

As in our previous work, the choice the test statistic is based the fact that the Fourier coefficients of the transformed empirical process wr to a given orthonormal basis are asymptotically standard Normal. Different combinations of Fourier coefficients allow the identification of different alternatives.

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A New Framework for Distance and Kernel-based Metrics in High Dimensions

Shubhadeep Chakraborty,[1] *Department of Statistics, Texas A&M University*
Xianyang Zhang, *Department of Statistics, Texas A&M University*

Abstract. The paper presents new metrics to quantify and test for (i) the equality of distributions and (ii) the independence between two high-dimensional random vectors. We show that the energy distance based on the usual Euclidean distance cannot completely characterize the homogeneity of two high-dimensional distributions in the sense that it only detects the equality of means and the traces of covariance matrices in the high-dimensional setup. We propose a new class of metrics which inherit the desirable properties of the energy distance and maximum mean discrepancy/(generalized) distance covariance and the Hilbert-Schmidt Independence Criterion in the low-dimensional setting and is capable of detecting the homogeneity of/completely characterizing independence between the low-dimensional marginal distributions in the high dimensional setup. We further propose t-tests based on the new metrics to perform high-dimensional two-sample testing/independence testing and study their asymptotic behavior under both high dimension low sample size (HDLSS) and high dimension medium sample size (HDMSS) setups. The computational complexity of the t-tests only grows linearly with the dimension and thus is scalable to very high dimensional data. We demonstrate the superior power behavior of the proposed tests for homogeneity of distributions and independence via both simulated and real datasets.

Keywords: Distance Covariance, Energy Distance, High Dimensionality, Hilbert-Schmidt Independence Criterion, Independence Test, Maximum Mean Discrepency, Two Sample Test, U-statistic.

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Spatial sampling and entropy

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Abstract. Many traits of environmental sampling are associated to spatial sampling. Indeed, the expression "environmental sampling" characterizes different general situations: under the reference population viewpoint, the focus is on natural populations, with stress on biological or pollution situations, while, under the sampling perspective, spatial sampling is a very important option. The link between sampling and entropy has been extensively debated. Maximum entropy sampling is a topic that has been intensively deepened, especially in computer science, where it received important contributions. Under a different framework, a sequel of papers pointed out that the usual computation of the entropy of a random variable does not take space into account and that the solutions proposed to face this peculiarity are more sophisticated adhockeries rather than strict solutions to the general problem of computing spatial entropies. In these works, we considered 4 basic spatial configurations named "compact", "repulsive", "cluster" and "random", noticing that the traditional entropy assumes the same value for any of them. Considering the spatial configuration as an auxiliary information is of help for more precise estimation of the probabilities that enter in the entropy. The variable of interest can be transformed in order to obtain the suitable joint manifestation. An important challenge is therefore to deepen how the foundations of spatial sampling influence entropy computation. A basic point to remember is that, in any developments, the correct definition of the support of the random variable whose probabilities have to be computed as additive terms of an entropy is very important. Since the spatial structure of a population influences the value of entropy, this works investigates the relationships between the ascertainment of a spatial structure on the population and spatial sampling. The accurate definition of the random variable to consider and of their support are fundamental. The distances between values of the auxiliary variable are also very useful.

Keywords: Environmental sampling, Spatial sampling, Spatial entropy

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Model Checks for Marginal Effects in Proportional Hazard Models

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Abstract. The article considers a specification test of the parametric part of proportional hazard models, which determines the covariate effects. Wrong specification consists of misspecification of the functional form of the covariates and misspecification of the link function. The test is based on a CUSUM process of the martingale residuals that are obtained from the Doob-Meyer decomposition and evaluated at the partial-likelihood estimator and the Breslow estimator. We develop principal component decomposition (PCD) of the CUSUM residual process. The obtained components, which are asymptotically distributed as independent standard normal variables, provide a basis for different types of goodness-of-fit tests that specialized in certain directions. The PCD method we propose is based on the landmark paper by Durbin, Knott and Taylor (1975), however, we extend their method, which only works for parametric efficient estimator, in such a way that it is able to accommodate any $n^{1/2}$-consistent estimator of both parametric and nonparametric functions. This is motivated from the fact that in the proportional hazard models, usually not only some finite-dimensional regression parameters need to be estimated, but also a fully unspecified baseline hazard function, hence the existing PCD method does not apply. The PCD method in this article is applicable to goodness-of-fit test of a general class of duration models. As a result, the omnibus Cramér-von Mises test, which is the squared L^2-norm of the CUSUM process, has an orthogonal representation as a weighted sum of all squared components. Smooth tests that based on a few components are also constructed. Finite sample performance of the proposed tests is illustrated in the context of a Monte Carlo experiment.

Keywords: Duration analysis, Goodness-of-fit, Principal component decomposition, Cramér-von Mises, Right-censorship

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A Pearson's Independence Test for Conditional Distribution Model Checking

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Abstract. A new goodness-of-fit test for continuous conditional distributions, based on the Pearson type test of independence, is proposed. The test exploits the fact that, under a correct specification, the conditional probability integral transform of the explained variable is independent of the explanatory variables. Unlike existing Pearson's tests for conditional distributions, the test statistic proposed is distributed as a chi-square with known degrees of freedom when using general partitions that may depend on the sample. We propose alternative data grouping algorithms in multiple dimensions to construct partitions for constructing tests with different properties. The finite sample performance of the test is investigated by means of Monte Carlo simulations.

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Testing relevant hypotheses in functional time series via self-normalization

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Abstract.

In this paper we develop methodology for testing relevant hypotheses in a tuning-free way. Our main focus is on functional time series, but extensions to other settings are also discussed. Instead of testing for exact equality, for example for the equality of two mean functions from two independent time series, we propose to test a *relevant* deviation under the null hypothesis. In the two sample problem this means that an L^2-distance between the two mean functions is smaller than a pre-specified threshold. For such hypotheses self-normalization, which was introduced by Shao (2010) and Shao and Zhang (2010) and is commonly used to avoid the estimation of nuisance parameters, is not directly applicable. We develop new self-normalized procedures for testing relevant hypotheses and demonstrate the particular advantages of this approach in the comparisons of eigenfunctions.

Keywords: self normalization, functional time series, two sample problems, change point analysis, CUSUM, relevant hypotheses

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Robust Model Checks with High-Dimensional Covariates

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Abstract. The presence of high-dimensional covariates makes classical model checks for econometric models not applicable when parameters are estimated by regularized methods such as the Lasso. Furthermore, the use of such nonlinear estimators makes commonly used resampling methods very time consuming, particularly so for big data applications. This paper proposes omnibus tests for conditional moment restrictions of stationary time series based on continuous functionals of a Neyman-orthogonalized projected integrated conditional moment process. This methodology leads to new tests for many models of interest, having the following remarkable properties: (i) they do not need to choose a tuning parameter; (ii) there is no need for re-estimating the unknown parameters in each bootstrap replication; (iii) the new tests are valid under higher order conditional moments of unknown form; and (iv) they allow for regularized root-n estimators that are not asymptotically linear (e.g Lasso). A Monte Carlo experiment shows that the new method presents more accurate size and higher power than competing procedures. In an empirical application we study the dynamics in mean and variance of the Hong Kong stock market index and we evaluate models for the Value-at-Risk of the S&P 500. These applications highlight the merits of our approach and show that the new methods have higher power than popular backtesting methods.

Keywords: Conditional moment restrictions, omnibus tests, asset pricing, Value-at-Risk, Expected Shortfall, backtesting

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On projection-based tests of uniformity on the hypersphere

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Juan A. Cuesta-Albertos, *Department of Mathematics, Statistics and Computation, University of Cantabria*

Abstract. Testing uniformity of a sample supported on the hypersphere is one of the first steps when analyzing multivariate data for which only the directions (and not the magnitudes) are of interest. In this work, a projection-based class of uniformity tests on the hypersphere is introduced. The new class allows for extensions of circular-only uniformity tests and introduces the first instance of an Anderson–Darling test in the context of directional data. A simulation study corroborates the theoretical findings. Finally, a real data example illustrates the usage of the new tests.

Keywords: circular data, directional data, hypersphere, Sobolev tests, uniformity

1 SETTING

Testing uniformity of a sample $\mathbf{X}_1, \ldots, \mathbf{X}_n$ of a random vector $\mathbf{X}$ supported on the hypersphere $\Omega_q := \{\mathbf{x} \in \mathbb{R}^{q+1} : \mathbf{x}'\mathbf{x} = 1\}$ of $\mathbb{R}^{q+1}$, with $q \geq 1$ is one of the first steps when analysing multivariate data for which only the directions (and not the magnitudes) are of interest – the so-called *directional data*. This kind of data arise in many applied disciplines, such as astronomy, biology, etc.

The inspiration for this contribution comes from the projection-based test of Cuesta-Albertos et al. (2009), which is based on the fact that the distribution of $\mathbf{X}$ is determined by that of a one-dimensional *random* projection, $\boldsymbol{\gamma}'\mathbf{X}$. For each $\boldsymbol{\gamma}$ (uniformly distributed on Ω_q and independent of the sample), Cuesta-Albertos et al. (2009) considered a Kolmogorov–Smirnov test statistic on the projected sample $\boldsymbol{\gamma}'\mathbf{x}_1, \ldots, \boldsymbol{\gamma}'\mathbf{x}_n$. This test clearly depends on $\boldsymbol{\gamma}$, which Cuesta-Albertos et al. (2009) mitigates by taking k random

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directions $\boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_k$ and combining the p-values associated to each of the k tests.

2 RESULTS

Differently from Cuesta-Albertos et al. (2009), we consider for each $\boldsymbol{\gamma}$ the well-known weighted quadratic norm by Anderson and Darling (1954):

$$Q^w_{n,q,\boldsymbol{\gamma}} := n \int_{-1}^{1} \left(F_{n,\boldsymbol{\gamma}}(x) - F_q(x)\right)^2 w(F_q(x))\, \mathrm{d}F_q(x), \tag{1.2}$$

where w is a weight function, $F_{n,\boldsymbol{\gamma}}$ and F_q are the empirical cumulative distribution function and the cumulative distribution function of the projected sample, respectively. In addition, instead of drawing several random directions and aggregating afterwards the outcomes of the associated tests, our statistic itself gathers information from all the directions on Ω_q: it is defined as the *expectation* of (1.2) with respect to $\boldsymbol{\gamma}$. The new class of uniformity tests is thus the one indexed by the weights w.

Using this formulation, simple expressions for several test statistics are obtained for the circle and sphere, and relatively tractable forms for higher dimensions. Despite their different origins, the proposed class and the well-studied Sobolev class of uniformity tests (see Prentice (1978)) are shown to be related. Our new parametrization proves itself advantageous by allowing to derive new tests for hyperspherical data that neatly extend the circular tests by Watson, Ajne, and Rothman, and by introducing the first instance of an Anderson–Darling-like test in such context. The asymptotic distributions and the local optimality against certain alternatives of the new tests are obtained.

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Exceedance locations in nonparametric regression under Gaussian subordination

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Abstract. We consider estimation of locations where a nonparametric regression function exceeds a given threshold. Of special interest are regression errors that are unknown transformations of a latent Gaussian process. We discuss asymptotic results and data examples to motivate discussions.

Keywords: Change points, Exceedance, Gaussian subordination, Kernel smoothing, Spatial data

1 INTRODUCTION

Estimating locations and extent of threshold exceedance have important applications in many sciences. For time series data, exceedances of the first derivative of the trend function beyond a given threshold may be used to define locations of rapid changes in the trend function. This is considered in Menendez et al. (2010) for a Gaussian subordinated long-memory series, where the authors illustrate their method by estimating rapid climate change points in an oxygen isotope series from Greenland. These climate change points can be found around the time of what is known as the *Younger Dryas* (Alley, 2000), more than 10,000 years before present.

In case of spatial observations, the local modes or spatial locations where the mean surface exceeds a given threshold are of interest in various fields. In forestry, locations with potentially large volumes of timber may be formulated as exceedance locations (Moser, 2014). In the earth's atmosphere, regions where the total column ozone is low (Rowland, 2006) may be defined in terms of non-exceedance locations with respect to a predefined threshold. For such data, a lattice model with long memory is considered in Beran et al. (2009). In this talk, we consider a nonparametric regression model for spatial data and address exceedance location estimation. Of special interest are

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observations that are subordinated to a long-memory Gaussian process. For the regression surface estimation, we apply kernel smoothing (Ghosh, 2015). For a partial review of nonparametric curve and surface estimation under Gaussian subordination see Ghosh (2018), whereas an extended coverage of statistical inference for processes with long-range dependence is in Beran et al. (2013).

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A goodness-of-fit test for the functional linear model with functional response

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Abstract. Functional data analysis enables to exploit the complexity and richness of data measured over continuous domains. When two functional random variables are available, it may be useful to determine their relation by means of a regression model. If the regression function is a linear Hilbert–Schmidt operator between two L^2 spaces, we are under the functional linear model with functional response. We propose a novel goodness-of-fit test for the null (composite) hypothesis of this model, against a general, unspecified alternative, leading to an omnibus test. The test statistic is formulated in terms of the quadratic norm over a doubly-projected empirical process, and is easy to compute, interpret and calibrate on its distribution via a wild bootstrap on the residuals. A flexible hybrid approach involving LASSO regularization and linearly-constrained least-squares is used to perform the selection of the number of dimensions when estimating the residuals. The finite sample behaviour of the test, regarding power and size, is illustrated via a complete simulation study under varying scenarios. The test is applied to several real datasets to check the validity of the model.

Keywords: Cramér–von Mises statistic; functional data analysis; functional linear model; functional response; Goodness-of-Fit omnibus test; LASSO; wild bootstrap.

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Structural Stability of Infinite-Order Regression

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Abstract. We develop a class of tests for the structural stability of infinite order regression models, when the time of a structural change is unknown. Examples include the infinite order autoregressive model, the nonparametric sieve regression and many others whose dimension grow to infinity. Allowing for dependent data, we approximate our infinite-dimensional testing problem with a sequence of finite-dimensional problems of increasing dimension.

When the number of parameters diverges, the traditional tests such as the supremum of Wald, LM or LR statistic or their exponentially weighted averages diverge as well. However, we show that a suitable transformation of these tests converges to a proper weak limit as the sample size n and the dimension p grow to infinity simultaneously. In general, this limit distribution is different from the sequential limit, which can be obtained by increasing the order p of the standardized tied-down Bessel process in Andrews (1993). Most interestingly, our joint asymptotic analysis discovers that the joint asymptotic distribution depends on a higher order serial correlation.

We also establish a weighted power optimality property of our tests under certain regularity conditions. However, the theory must take into account, once again, the infinite-dimensional aspect of the problem. In particular, we need to carefully define our notion of optimality as, in general, optimality properties typically require a finite number of directions in which the null may be violated. We adopt the spirit of the sieve regression and assume that we can direct our test to have reasonable power against the increasing number of the approximating directions and the remaining directions can be neglected. Among those approximating directions, we assign weights inspired by Andrews and Ploberger (1994).

A new result on partial sums of random matrices is established, which may be of independent interest. This may be viewed as a complement to existing results for sums of random matrices, see e.g. Chen and Christensen (2015).

We examine finite-sample performance in a Monte Carlo study, in particular illustrating the consequences of ignoring higher order serial correlation. A number of empirical examples apply the test to real world datasets, and include both nonparametric models as well as AR(p) models with large p.

Keywords: Structural change, nonparametric regression, unknown change point, optimal test, infinite-order autoregression, high order serial correlation.

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Testing multivariate normality by zeros of the harmonic oscillator in characteristic function spaces

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Abstract. We study a novel class of affine invariant tests for multivariate normality. Writing $\varphi(t) = \exp(-\|t\|^2/2)$ for the characteristic function of the d-variate standard normal distribution, these tests are based on the fact that the function φ is the only solution of the partial differential equation

$$\begin{cases} \Delta\varphi(x) = (\|x\|^2 - d)\varphi(x), & x \in \mathbb{R}^d, \\ \varphi(0) = 1, \end{cases} \tag{1.3}$$

where Δ stands for the Laplace operator. The operator $(-\Delta + \|x\|^2 - d)$ is known as the harmonic oscillator. For $d = 1$, equation (1.3) reduces to a fixed point problem for the Hermite operator.

Suppose $X_1, X_2, \ldots$ is a sequence of i.i.d. d-variate random (column) vectors with a distribution that is assumed to be absolutely continuous with respect to Lebesgue measure. Let $\overline{X}_n = n^{-1}\sum_{j=1}^n X_j$ and $S_n = n^{-1}\sum_{j=1}^n (X_j - \overline{X}_n)(X_j - \overline{X}_n)^\top$ denote the sample mean and the sample covariance matrix of $X_1, \ldots, X_n$, respectively, and write $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $j = 1, \ldots, n$ for the scaled residuals, where $S_n^{-1/2}$ is the symmetric square root of S_n^{-1} (which exists with probability one if $n \geq d+1$). Furthermore, let

$$\varphi_n(t) = \frac{1}{n}\sum_{j=1}^n \exp\left(\mathrm{i}t^\top Y_{n,j}\right), \qquad t \in \mathbb{R}^d,$$

be the empirical characteristic function of $Y_{n,1}, \ldots, Y_{n,n}$. To test the hypothesis H_0 that the distribution of X_1 is some non-degenerate d-variate normal distribution, we propose the weighted L^2-statistic

$$T_{n,a} = n\int_{\mathbb{R}^d} \left|\Delta\varphi_n(t) - (\|t\|^2 - d)\varphi(t)\right|^2 \exp\left(-a\|t\|^2\right) \mathrm{d}t,$$

where $a > 0$ is a suitable tuning parameter. With the weight function $\exp\left(-a\|t\|^2\right)$, the test statistic $T_{n,a}$ allows for a simple expression that does not involve any integration and is thus amenable for computational purposes. We derive the limit null distribution of $T_{n,a}$ as $n \to \infty$ and show the consistency of a test of H_0 that rejects H_0 for large values of $T_{n,a}$ against general alternatives. As $a \to \infty$, a rescaled version of $T_{n,a}$ converges to a measure of multivariate skewness due to Móri, Rohatgi and Székely. The test shows strong empirical power with respect to prominent competitors.

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Keywords: test for multivariate normality, empirical characteristic function, harmonic oscillator

Goodness-of-fit test for (log-) normality based on a derivative property of the log-normal Laplace transform

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Abstract. We propose a new goodness-of-fit for composite hypothesis of univariate (log-) normality using a relation between the log-normal Laplace transform and its derivative. Power assessments and promising comparisons with numerous other tests are obtained in an extensive Monte Carlo experiment.

Keywords: empirical process, empirical Laplace transform, goodness-of-fit test, normal distribution, parametric bootstrap

Goodness-of-fit tests based on empirical integral transforms belong already to well recognized inferential procedures. In particular, testing for normality in this way continue to receive attention. In addition to the early test by Epps and Pulley (1982) a couple of new ones have been introduced more recently; see e.g. Zghoul (2010), Meintanis et al. (2014), Henze et al. (2019).

Here, yet another method based on the Laplace transform of the log-normal(μ,σ) density is proposed. An explicit form of this transform in not available but we can show that it satisfies the following differential equation $L'_{\mathrm{X}}(s) = -e^{\mu+\sigma^2/2}L_{\mathrm{X}}(se^{\sigma^2})$, which seems to be unknown in literature; see e.g. Asmussen et al. (2016).

Given an i.i.d. sample $(X_1, ...X_n)$, the composite hypothesis about its log-normality may be tested using the following weighted L_2-type statistic

$$T_n = n\int_0^\infty \left[\widehat{L}'_{\mathrm{Y}}(s) + e^{1/2}\widehat{L}_{\mathrm{Y}}(se)\right]^2 w(s;a)\mathrm{d}s\,, \tag{1.4}$$

where the transform and its derivative are replaced by their empirical counterparts computed from the standardized sample $(Y_1, ..., Y_n)$ and $Y_i = \exp\{(\log(X_i) - \widehat{\mu})/\widehat{\sigma}\}$ with some consistent estimators of μ and σ (for testing for normality we use $Y_i = \exp\{(X_i - \widehat{\mu})/\widehat{\sigma}\}$). The parameterized decaying

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weight function $w(s;a)$ controls the power of the test against specific alternatives and choice of the exponential function e^{-as} leads to an easily computable expression for (1.4).

We discuss the issues of test's consistency and the distribution of test statistic. As this distribution is difficult to derive, we resort to the parametric bootstrap to obtain the critical values in the implementation.

Monte Carlo simulations are conducted to assess the power in case of a broad range of symmetric and skewed alternatives for both the normal and log-normal null hypotheses. Comparisons with three groups of other tests are made, namely with i) the classical omnibus tests: Kolmogorov-Smirnov, Cramér-von Mises and Anderson-Darling; ii) the popular and powerful moment-based tests for normality: Shaphiro-Wilk and Jarque-Bera; iii) the other aforementioned transform-based tests. The proposed test definitely outperforms the procedures from group i), while when it comes to those from ii) and iii) it is comparable or sometimes better provided the parameter of the weight function is chosen appropriately. These characteristics with respect to other procedures hold almost uniformly over sample sizes from $n = 20$ to 140 (which is not always the case for the transform-based tests).

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How to Distinguish Abrupt Structural Breaks from Smooth Structural Changes?

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Abstract.

Structural changes often occur in economics and finance due to changes in preferences, technologies, institutional reforms, policies, crises and other factors. It is important to distinguish whether a structural change is abrupt or evolutionary, because the implications on econometric modelling and inferences will be different. In this paper, we propose a nonparametric consistent test for the null hypothesis of smooth structural changes against the alternative hypothesis of at least an abrupt structural break. The basic idea is to check whether smoothed nonparametric estimators for the left and right limits of time-varying parameters in a linear regression model converge to the same limit at every time point in the sample period. We avoid making any restrictive assumption on a parametric functional form for time-varying parameters, and allow to use data-driven time-varying bandwidths in nonparametric estimation. We derive the asymptotic nonstandard null distribution of a test based on the maximal difference between the left and right limit estimators and establish the consistency of the test under the alternative. A Hansen's-type (1996) resampling method is used to compute the p-value of the test statistic. Our simulation study highlights the merits of the proposed test. In an empirical application, we find strong evidence in favor of existence of abrupt structural breaks for most predictive regressions of equity returns, and the dates of these structural breaks are closely related to important historical macroeconomic events and policy changes.

Keywords: Abrupt structural break, Boundary problem, Local linear smoothing, Model instability, Predictive regression models, Smooth structural change

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Testing for hidden periodicities in functional time series

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Abstract. Periodicity is one of the most important characteristics of time series, and tests for periodicity go back to the very origins of the field. We consider the situation where the potential period of a multivariate or functional (i.e. infinite dimensional) time series (FTS) is unknown and hence extend the recent paper Hörmann et al. (2018) where a fixed period was considered. As a test statistic we consider the maximum norm of the multivariate and functional periodogram over fundamental frequencies. The main theoretical problem is to derive the limiting distribution of this test statistic for a functional random sample. To this end we first tackle the multivariate case and show that under some moment restrictions it is in the domain of attraction of the Gumbel distribution. To the best of our knowledge this result is the first multivariate extension of a famous result of Davis and Mikosch (1999). We also provide a further extension to linear processes following the classical approach of Walker (1965). In a second step we pass from multivariate to infinite dimensional data. We follow a common approach in FDA, which is to project the functional data on the first $p = p_n$ eigenfunctions (principal components) of the underlying covariance operator and then let $p \to \infty$. This step is technically quite delicate. The main ingredient of our proof is based on a Berry-Esseen type result from Chernozhukov et al. (2017) over sparsely convex sets. This result in turn relies on a powerful anti-concentration inequality due to Nazarov (2003). We illustrate our approach by a small numerical experiment and show that the finite sample performance is very satisfactory. Moreover, the approach gives a very reliable estimate for the unknown length of the period.

Keywords: functional data, high dimensional inference, periodogram, time series

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Inference in dynamic Nelson–Siegel models

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Abstract. We discuss several Nelson–Siegel type dynamical models. We suggest using the least square method to estimate the parameters. We provide an application of the main result to discuss the validity of the Nelson–Siegel model for the data.

Keywords: Factor model, Term structure, Hypothesis testing

1 INTRODUCTION

We consider functional observations $X_1(t), X_2(t), \ldots, X_N(t)$ defined on a compact set $\mathcal{T}$, which may be an interval or a discrete set. We assume that these observations follow the model

$$X_i(t) = \sum_{\ell=1}^{K} b_{i,\ell,0} f_\ell(t; \boldsymbol{\lambda}_0) + \epsilon_i(t), \text{ with } E\epsilon_i(t) = 0, \ t \in \mathcal{T}, \ 1 \le i \le N, \tag{1.5}$$

where the random coefficients satisfy

$$b_{i,\ell,0} = c_{\ell,0} + e_{i,\ell} \text{ with, } Ee_{i,\ell} = 0, \ 1 \le \ell \le K \text{ and } 1 \le i \le N. \tag{1.6}$$

Under assumptions (1.5) and (1.6)

$$EX_i(t) = \sum_{\ell=1}^{K} c_{\ell,0} f_\ell(t; \boldsymbol{\lambda}_0), \ t \in \mathcal{T} \text{ and } 1 \le i \le N,$$

i.e. the mean of the observations can be written as a linear combination of the functions $f_1(t; \boldsymbol{\lambda}_0), f_2(t; \boldsymbol{\lambda}_0), \ldots, f_K(t; \boldsymbol{\lambda}_0)$, where the functions $f_1, f_2, \ldots, f_K$ are known and $\boldsymbol{\lambda}_0 \in R^d$ is the true value of an unknown parameter. In most current applications λ_0 is fixed and set to a value based on previous experience. It is generally not estimated, so the curves $f_k(t; \boldsymbol{\lambda}_0)$ are treated as given, deterministic functions. We will call these functions functional

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factors, or simply factors. Our exposition is motivated by the Nelson–Siegel model (cf. Diebold and Rudebusch, 2013) and its extensions. In the classical Nelson and Siegel (1987) model for instantaneous forward rate, there is only one parameter $\lambda_0 \in R$, which determines three factors:

$$f_1(t;\lambda_0) = 1, \quad f_2(t;\lambda_0) = \frac{1-e^{-\lambda_0 t}}{\lambda_0 t} \quad \text{and} \quad f_3(t;\lambda_0) = \frac{1-e^{-\lambda_0 t}}{\lambda_0 t} - e^{-\lambda_0 t},$$

$0 \le t \le 1$. The function f_1 is usually referred as “level", f_2 and f_3 are called “slope" and "curvature". We are interested in the estimation of the parameter vector

$$\mathbf{a} = (c_1, c_2, \ldots, c_K, \boldsymbol{\lambda}^{\mathrm{T}}) \in R^{K+d} \tag{1.7}$$

whose true value is $\mathbf{a}_0 = (c_{1,0}, c_{2,0}, \ldots, c_{K,0}, \boldsymbol{\lambda}_0^{\mathrm{T}})$. Our methodology and theory can accommodate heteroskedastic functional errors ϵ_i in (1.5) and heteroskedastic multivariate errors $e_{i,\ell}$ in (1.6) generalizing Bardsley et al. (2017).

Our approach consists in minimizing the least squares loss function

$$U_N(\mathbf{a}) = \frac{1}{N}\sum_{i=1}^{N}\int\left(X_i(t) - \sum_{\ell=1}^{K} c_\ell f_\ell(t;\boldsymbol{\lambda})\right)^2 dt$$

with $\mathbf{a}$ given by (1.7) The estimator $\hat{\mathbf{a}}_N$ is thus defined by

$$\hat{\mathbf{a}}_N = \mathrm{argmax}_{\mathbf{a}\in\mathbf{A}} U_N(\mathbf{a}). \tag{1.8}$$

We prove the asymptotic normality of $N^{1/2}(\hat{\mathbf{a}}_N - \mathbf{a}_0)$ and also discuss its application to test if the Nelson–Siegel model fits the data.

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Tests for discrete valued time series based on probability generating functions

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Abstract. Time series of counts enjoy numerous applications in various fields as business, economics, engineering, epidemiology, and others. Hence, there has been a growing interest in studying such time series in recent years, and various models (univariate as well as multivariate) have been proposed in the literature. As a number of various models is available, there is naturally a need for goodness-of-fit tests. We deal with a null hypothesis that a given data follow a particular model for time series of counts. Special emphasis is given to the popular models of integer autoregression (INAR) and Poisson autoregression (INARCH) and their bivariate generalizations, but the procedure can be generalized to other models in a straightforward way. Our goodness-of-fit test statistics is based on the empirical probability generating function (PGF). More specifically, it is a weighted L^2 distance between the ordinary empirical probability generating function and a semiparametric estimator of PGF, which uses the structure and properties of the model. The asymptotic behavior of the test statistics is investigated. As the asymptotic distribution is complicated and depends on the unknown parameters in a complex way, a bootstrap version of the test is proposed. The finite–sample properties of the proposed procedure is illustrated by a simulation study and a real data example.

Keywords: goodness-of-fit test, probability generating function, integer valued time series, Poisson distribution

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Monitoring procedures in two-sample multivariate situations

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Abstract. The talk concerns monitoring (sequential, on-line) procedures in two-sample multivariate setup. The proposed tests are L2-type criteria based on empirical characteristic functions. Asymptotic properties are presented together with results of simulation study. The developed method is applied on a real data-set from the financial sector over a time period which includes the Brexit referendum.

Keywords: change-point, monitoring, empirical characteristic function, two-sample problem

1 INTRODUCTION

Change point detection both off-line and on-line versions is considered for long time both because of practical applications and interesting theoretical problems in connection with investigation of properties of developed procedures. So far the main interest was in what could be called single-sample variant.

In the talk we formulate two-sample change detection situations and propose monitoring procedures with finite-dimensional independent observations as well as with multidimensional time series. A number of various test procedures can be developed for this problem, e.g. CUSUM type procedures along the lines of single sample variants. Here the suggested procedures employ functionals of empirical characteristic functions that has appeared quite reasonable type of statistics also in other statistical problems (single-sample change point problem, goodness-of-fit tests).

The talk is mostly based on papers Hlávka and Hušková (2018) and Hlávka, Hušková and Meintanis (2019).

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2 FORMULATION AND PROCEDURES

Particularly, we consider a stationary time series $(x_t, y_t), t = 1, 2, ...,$ where x_t and y_t are p-dimensional. The mode of dependence between the sequences x_t, and $y_t, t = 1, 2, ...,$ can be rather general. The focus is on monitoring (on-line) detection procedures.
The null hypothesis is formulated as

$$H_0 : F_t \equiv G_t \equiv K_0, \text{ for all } t = 1, 2, ...,$$

where F_t (resp. G_t) denote the marginal distribution function of $\boldsymbol{x}_t$ (resp. $\boldsymbol{y}_t$), and K_0 is an unknown fixed distribution function. The alternative hypothesis

$$H_1 : F_t \equiv G_t \equiv K_0, \ t \leq t_0, \ F_t \equiv K^{(F)}, \ G_t \equiv K^{(G)}, \ t > t_0,$$

where t_0 is a change point, and $K^{(F)} \neq K^{(G)}$, with either $K^{(F)} \neq K_0$ or $K^{(G)} \neq K_0$, all quantities being unknown.
The considered test statistic is defined as

$$D_{t,w} = D_{t,w}(\widehat{\varphi}_{\boldsymbol{x},t}, \widehat{\varphi}_{\boldsymbol{y},t}) = \int_{\mathbb{R}^p} |\widehat{\varphi}_{\boldsymbol{x},t}(\boldsymbol{u}) - \widehat{\varphi}_{y,t}(\boldsymbol{u})|^2 \, w(\boldsymbol{u}) d\boldsymbol{u}, \quad t = 1, 2, \ldots,$$

where

$$\widehat{\varphi}_{\boldsymbol{x},t}(\boldsymbol{u}) = \frac{1}{t} \sum_{\tau=1}^{t} e^{i\boldsymbol{u}^\top \boldsymbol{x}_\tau}, \qquad \widehat{\varphi}_{\boldsymbol{y},t}(\boldsymbol{u}) = \frac{1}{t} \sum_{\tau=1}^{t} e^{i\boldsymbol{u}^\top \boldsymbol{y}_\tau}$$

are the empirical CFs computed from $\boldsymbol{x}_\tau$ and $\boldsymbol{y}_\tau$, $\tau = 1, ..., t$, respectively, and $\boldsymbol{u}^\top \boldsymbol{y}_\tau$ denotes the scalar product. The null hypothesis H_0 is rejected for large values of $D_{t,w}$. Since it is assumed that data arrive sequentially, the problem is whether to stop and decide either in favor of alternative or postpone decision and continue collecting observations.
The results for asymptotic behavior both under the null as well as alternative hypotheses will be presented. Results of simulation study together with application to financial sector which also provides motivation for considering this version of detection of changes.

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Generalized Spacings Estimators

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Abstract. Spacings, which are the gaps between successive observations, have been utilized in statistical inference both for estimation and in testing of hypotheses. After a brief review of this area, we introduce estimators based on higher-order or multi-step spacings, called the "Generalized Spacings Estimators (GSEs)". Such estimators are obtained by minimizing the so-called Csiszar divergence between the empirical and the true distributions. Maximum likelihood estimators (MLEs) can be viewed as a special case, and GSEs are clearly needed when the MLEs do not exist. Current results generalize much of the earlier work on spacings-based estimation. These estimators are shown to be consistent as well as asymptotically normal under quite general conditions. When the step size and the number of spacings grow with the sample size, an asymptotically efficient class of estimators, called the "Minimum Power Divergence Estimators," are shown to exist. Simulation studies show that these asymptotically efficient estimators, perform very well in finite samples relative to the MLEs, and unlike the MLEs, are quite robust even under heavy contamination.

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On the BHEP test for functional data

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Abstract. The objective of this paper is to extend the BHEP test for testing normality to the functional data context.

Keywords: functional data, normality, consistency.

1 INTRODUCTION

Many statistical procedures for finite dimensional data assume that the data is normally distributed. Because of this reason, a number of normality test have been proposed in the statistical literature. Some of them are based on certain properties characterizing the normality law, while others are based on comparing a nonparametric estimator of a function characterizing a probability law with a parametric estimator of that function, obtained under the null hypothesis. The test of Epps and Pulley (1983) for testing univariate normality, that was later extended to the multivariate case by Baringhaus and Henze (1988), belongs to the later class of tests. It is based on comparing the empirical characteristic function of the data with the characteristic function of the normal law. It is usually referred to as the BHEP test. Because of its nice properties, the BHEP test has been extended in several directions such as testing for normality of the errors in linear models (Jiménez-Gamero et al., 2005), in nonparametric regression models (Hušková and Meintanis, 2010) and in GARCH models (Jiménez-Gamero, 2014), just to cite a few.

The normality assumption is important not only in the classical context (from now on, by *classical* we refer to the case where the available data take values in $\mathbb{R}^d$, for some fixed $d \in \mathbb{N}$), but also in other settings such as functional data analysis. There is some inferential procedures designed for functional data that assume normality. Examples are the test for the equality of covariance operators in Ch. 5 of Horváth and Kokoszka (2012), or the test in Zhang et al. (2010) for the equality of means. On the other hand, some methods are valid under quite general assumptions, but they

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greatly simplifies when the normality assumption is added (see, for example, Boente et al., 2018). Therefore, when dealing with functional data, it is also of interest the problem of testing for normality.

Goodness-of-fit tests for the probability measure generating functional data have been proposed in the statistical literature. Cuesta-Albertos et al. (2007) have proposed a test based on random projections, Bugni et al. (2009) have studied an extension of the Cramér-von Mises test to infinite-dimensional random variables, and Ditzhaus and Gaigall (2018) have proposed a test that integrates, along all possible projections, all univariate Cramér-von Mises tests obtained by projecting the data. Since, as shown in Laha and Rohatgi (1979), the probability distribution of a random element taking values in a separable Banach space is characterized by its characteristic function, the objective of this paper is to extend the BHEP test to the functional data context.

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Bootstrap of residual processes in regression: to smooth or not to smooth?

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Abstract. In this paper we consider regression models with centered errors, independent from the covariates. Given independent and identically distributed data and given an estimator of the regression function, which can be parametric or nonparametric of nature, we estimate the distribution of the error term by the empirical distribution of estimated residuals. To approximate the distribution of this estimator, Koul and Lahiri (1994) and Neumeyer (2009) proposed bootstrap procedures based on smoothing the residuals before drawing bootstrap samples. So far it has been an open question whether a classical non-smooth residual bootstrap is asymptotically valid in this context. In this paper we solve this open problem, and show that the non-smooth residual bootstrap is consistent. We illustrate this theoretical result by means of simulations, that show the accuracy of this bootstrap procedure for various models, testing procedures and sample sizes.

Keywords: Bootstrap; Empirical distribution function; Kernel smoothing; Linear regression; Location model; Nonparametric regression

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A fast large-sample alternative to the parametric bootstrap in goodness-of-fit testing

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Abstract. The process comparing the empirical cumulative distribution function of the sample with a parametric estimate of the cumulative distribution function is known as the *empirical process with estimated parameters* and has been extensively employed in the literature for goodness-of-fit testing.

The simplest way to carry out such goodness-of-fit tests, especially in a multivariate setting, is to use a *parametric bootstrap*. Although very easy to implement, the parametric bootstrap can become very computationally expensive as the sample size, the number of parameters, or the dimension of the data increase.

An alternative resampling technique based on a fast *weighted bootstrap* is presented in this talk, and is studied both theoretically and empirically in several contexts.

The outcome of this work is a generic and computationally efficient *multiplier* goodness-of-fit procedure that can be used as a large-sample alternative to the parametric bootstrap. The underlying ideas will be illustrated both in the classical context of assessing the fit of a multivariate distribution and in the context of goodness-of-fit tests for copulas.

In the classical context of assessing the fit of a multivariate distribution, to approximately determine how large the sample size needs to be for the parametric and weighted bootstraps to have roughly equivalent powers, extensive Monte Carlo experiments were carried out in dimension one, two and three, and for models containing up to nine parameters. The computational gains resulting from the use of the proposed multiplier goodness-of-fit procedure are illustrated on trivariate financial data.

A by-product of this work is a fast large-sample goodness-of-fit procedure for the bivariate and trivariate t distribution whose degrees of freedom are fixed.

In the context of goodness-of-fit testing for copulas, large scale Monte Carlo experiments, involving six frequently used parametric copula families and three different estimators of the copula parameter, confirm that the proposed procedures provide a valid, much faster alternative to the corresponding parametric bootstrap-based tests. An application of the derived tests to the modeling of a well-known insurance data set is presented. The use of the multiplier approach instead of the parametric bootstrap can reduce the computing time from about a day to minutes.

Keywords: goodness-of-fit testing, parametric bootstrap, weighted bootstrap, copulas

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Detection of change points in the mean function of spatio-temporal curves

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Abstract. We will present methodology for detecting a change in the annual pattern of an environmental variable measured at fixed locations in a spatial region. Using a framework built on functional data analysis, we model observations as time series of curves available at many sites (one functional time series for each location). Each sequence is modeled as an annual mean function, which may change, plus a sequence of error functions, which are spatially correlated. The tests statistics extend the cumulative sum (CUSUM) paradigm to this more complex setting. Their asymptotic distributions are not parameter free due to the spatial dependence, but can be effectively approximated by Monte Carlo methods. The new methodology is applied to precipitation data.

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Change point tests in nonparametric time series models

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Abstract. We consider a nonparametric heteroscedastic time series regression model and suggest testing procedures to detect changes in the regression function or in the conditional variance function.

Let $(Y_t, X_t)_{t\in\mathbb{Z}}$ be a weakly dependent stochastic process following the regression model

$$Y_t = m_t(X_t) + U_t, \quad t \in \mathbb{Z}.$$

The covariate X_t may include finitely many lagged values of Y_t, for instance $X_t = (Y_{t-1}, \ldots, Y_{t-d})$ such that the model includes nonparametric autoregression. The unobservable innovations are assumed to fulfill $E[U_t|U_{j-1}, X_j, j \leq t] = 0$ almost surely. Our assumptions on the innovations are very weak; in particular heteroscedastic models are covered. Assuming $(Y_1, X_1), \ldots, (Y_n, X_n)$ have been observed, our first aim is to test the null hypothesis

$$H_0 : m_t(\cdot) = m(\cdot),\ t = 1, \ldots, n,$$

for the conditional mean function $E[Y_t|X_t = x] = m_t(x)$, $t \in \mathbb{Z}$, and some not specified function m not depending on the time of observation t. The test statistics are continuous functionals of a sequential marked empirical process indexed in s and z,

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{\lfloor ns \rfloor} (Y_i - \hat{m}_n(X_i)) w_n(X_i) I\{X_i \leq z\}$$

(with a nonparametric estimator $\hat{m}_n$ for m and some weight function w_n) and thus combine classical CUSUM tests with marked empirical process approaches well known from goodness-of-fit testing. Weak convergence to a Gaussian process is shown under the null hypothesis in the case of a strictly stationary process. We obtain very simple limiting distributions and distribution-free tests in the case of univariate covariates. In contrast to simple CUSUM type tests the new tests are consistent against general change point alternatives. To obtain tests that react sensitive only to changes in m in cases where also changes in the conditional variance function may appear, we suggest bootstrap versions of the tests.

Our second aim is to test for changes in the conditional variance function in a model as above where the regression function is stable in time. To this end a modification of the sequential marked empirical process can be considered.

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Statistical Inference for Sparse High Dimensional Time Series Models

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Abstract. Fitting sparse models to high dimensional time series is an important area of statistical inference. In this paper we consider sparse vector autoregressive models and develop appropriate bootstrap methods to infer properties of such processes. Our bootstrap methodology generates pseudo time series using a model-based bootstrap procedure which involves an estimated, sparsified version of the underlying vector autoregressive model. Inference is performed using so-called de-sparsified or de-biased estimators of the autoregressive model parameters. We derive the asymptotic distribution of such estimators in the time series context and establish asymptotic validity of the bootstrap procedure proposed for estimation and, appropriately modified, for testing purposes. In particular we focus on testing that groups of autoregressive coefficients equal zero. Our theoretical results are complemented by simulations which investigate the finite sample performance of the bootstrap methodology proposed. A real-life data application is also presented.

Keywords: Vector Autoregressions, Lasso, Bootstrap, Testing

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$M-$estimation inference for partially linear single-index models: an empirical likelihood approach

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Abstract. Partially linear single-index models represent a versatile tool to capture the relationship between response variables and possibly high-dimensional covariate vectors. The approximation of the response is given by the sum of a linear term and of a nonparametric link function of a second linear combination of covariates, usually called the index. This approximation is defined with respect to a loss function which characterizes a feature of the conditional law of the response given the covariates such as the conditional mean or the conditional median. In this paper we consider a general family of loss functions and investigate the corresponding partially linear single-index regression models, including mean, quantile, expectile and robust regressions. Except for imposing some moments to be finite, the conditional law of the error term is allowed to be general. For the inference, we adopt the empirical likelihood (EL) approach based on a class of moment conditions in which we plug-in estimates of the nuisance link function. We show the asymptotic pivotality of the likelihood ratio under weak high-level conditions. A simple data-driven choice of the tuning parameter for the estimation of the link function is provided. Several extensions are proposed, including the generalized EL and the case where dimension of the parameter vector grows with the sample size.

Keywords: Confidence interval, Semiparametric regression, Wilks Theorem

1 INTRODUCTION

The observations are realizations of some random covariate vectors $X \in \mathbb{R}^{d_X}$ and $W \in \mathbb{R}^{d_W}$ and of a response variable Y. The components of the covariate vectors could be continuous or discrete variables. Consider a loss function $\mathcal{L}(u; v) = L(u - v)$, $u, v \in \mathbb{R}$, with $L(\cdot)$ a nonnegative piecewise differentiable convex function such that $L(0) = 0$, and let $\xi(\cdot)$ be its piecewise derivative. The quadratic, quantile and expectile losses are some examples. We propose a general semiparametric partially linear single-index model (PLSIM)

$$Y = X^\top \theta_1 + h(W^\top \theta_2) + \varepsilon, \qquad \text{with} \qquad \mathbb{E}(\xi(\varepsilon) \mid X, W) = 0 \quad \text{a.s.}, \tag{1.9}$$

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where $\theta = (\theta_1^\top, \theta_2^\top)^\top \in \Theta =\subset \mathbb{R}^{d_\theta}$, $d_\theta = d_X + d_W$, are unknown parameters and $h(\cdot)$ is an unknown univariate real-valued function, allowed to depend on θ. Our framework covers many semi-parametric models, some of them already investigated in the literature (see, for instance, (Ma and He, 2016), (Zhao et al., 2017), (Zhu and Xue, 2006), (Boente et al., 2006)), and many others not yet considered. The purpose of our paper is to propose an empirical likelihood (EL) approach for inference in the model (1.9). See Owen (1990), Owen (2001), Qin and Lawless (1994).

2 PLUG-IN EMPIRICAL LIKELIHOOD FOR PLSIM

The general model (1.9) could be rewritten under the form of a conditional moment equation

$$\mathbb{E}(\rho(Z;\theta,h) \mid X, W) = 0 \quad \text{a.s.}, \tag{1.10}$$

where $Z = (Y, X^\top, W^\top)^\top$ and $\rho(Z;\theta,h) = \xi\left(Y - X^\top\theta_1 - h(W^\top\theta_2)\right)$. The first task is to transform the conditional moment equation (1.10) in an suitable, equivalent, unconditional moment equation. Let $\mathrm{EL}_n(\theta, h)$ the empirical likelihood ratio corresponding to these equivalent moment equations. Next, to account for the infinite-dimensional parameter $h(\cdot)$ we adopt the profiling approach. See, for instance, Severini and Wong (1992), Liang et al. (2010). Let $\widehat{h}$ be the estimator of h and let θ_0 be the true value of θ. One of our main results is the following version of Wilks' Theorem.

Theorem 1. *In the class of general PLSIM introduced in equation (1.9), using some suitable unconditional moment equations,* $-2\log \mathrm{EL}_n(\theta_0, \widehat{h}) \to_d \chi^2_{d_\theta - 1}$.

To build $\widehat{h}$ we use a kernel approach for which we provide a suitable data-driven bandwidth rule that is easy to implement and perform quite well in applications.

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A new class of change point test statistics of Rényi type

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Abstract. A new class of change point test statistics is proposed that utilizes a weighting and trimming scheme for the cumulative sum (CUSUM) process inspired by Rényi (1953). A thorough asymptotic analysis and simulations both demonstrate that this new class of statistics possess improved power compared to traditional change point statistics based on the CUSUM process when the change point is near the beginning or end of the sample. Generalizations of these "Rényi" statistics are also developed to test for changes in the parameters in linear and non-linear regression models, and in generalized method of moments estimation. In these contexts we applied the proposed statistics, as well as several others, to test for changes in the coefficients of Fama-French factor models. We observed that the Rényi statistic was the most effective in terms of retrospectively detecting change points that occur near the endpoints of the sample.

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Permutation testing for goodness of fit and stochastic ordering

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Abstract. Problems of testing for ordered categorical variables are of great interest in many application disciplines, where a finite number of $Q \geq 1$ of such variables are observed on each individual unit (Pesarin and Salmaso (2006) (Pesarin and Salmaso, 2010a) and (Pesarin and Salmaso, 2010b)). In particular, Goodness of Fit tests are used to measuring how well do the observed data correspond to the assumption model. Several parametric solutions to univariate case have been proposed in literature. In particular, when dealing with categorical variables, the most used methods are Pearson's Chi-squared and Deviance statistic. However, these methods, usually based on the restricted maximum likelihood ratio test, are generally criticized because their asymptotic null and alternative distributions are mixtures of chi-squared variables whose weights essentially depend on underlying population distribution F and so the related degree of accuracy is difficult to assess and to characterize; thus their use when F is unknown is somewhat questionable in practice. Moreover, is well known the difficulty or impossibility to use them in multivariate cases. In many situations it can be of interest testing for a set of restricted alternatives to H_0 (Kim and Foutz (1997) and Chapman (1958)). In these cases we can refer to Stochastic Ordering. Parametric solutions don't allow this kind of tests. By working within the Non-parametric combination of dependent permutation tests, it is possible to find exact solutions to these problems. The NPC approach works as a general methodology for most multivariate situations, as for instance in cases where sample sizes are smaller than the number of observed variables, or where there are non-ignorable missing values, or when some of the variables are categorical (ordered and nominal) and others are quantitative and in many other complex situations. In this work, NPC tests for stochastic dominance are presented, both for two sample directional testing and for testing for a stochastic ordering in a multivariate setting. A simulation study is reported to show the NPC approach efficacy.

Keywords: Goodness of Fit, GOF, Stochastic ordering, NPC, Permutation test

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Multivariate Change-Point Asymptotics for Covariance Matrices of High-Dimensional Time Series

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Abstract. We study the problem to test for the presence of a change-point in a high-dimensional time series. The methodology is based on CUSUM statistics of bilinear forms of the sample covariance matrix. Asymptotic results are presented for a single statistic as well as a multivariate CUSUM transform where the dimension is allowed to grow as the sample sizes increases. Applications and relations to vector autoregressions and spiked covariance models are discussed. Finite sample properties are studied by simulations. The approach is illustrated by analyzing spatial-temporal data from ozone monitoring stations.

Keywords: Change-point, CUSUM, linear process, strong approximation, vector autoregression

The problem to test for a change in the covariance structure of a high-dimensional time series arises in diverse fields such as environmetrics, statistical genetics, finance or industrial quality control. Galeano and Peña (2007) studied the problem for a parametric Gaussian time series when the covariance matrix after the change is known. Aue et al. (2009) studied CUSUM procedures for fixed dimension and provided an approximation when d is large. This talk presents a methodology based on bilinear forms with respect to weighting vectors, which allows for analyses based on fixed bases (such as wavelets), random projections (have in mind the Johnson-Lindenstrauss theorem) or eigenstructures, Steland (2019). The assumed linear framework covers wide classes of time series, especially VARMA and spiked covariance models. VARMA models are heavily used in econometrics as well as environmetrics. Spiked covariance models have received considerable attention, in order to study statistical methods and their properties for high-dimensional data, especially to study the eigenstructure of covariance matrices, see e.g. Johnstone and Lu (2009), Cai et al. (2015) or Yata et al. (2018).

The statistical procedures cover unweighted and weighted cumulated sum (CUSUM) statistics and associated change-point estimators, multivariate CUSUM transforms of growing dimension, in order to handle analyses in

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subspaces, and χ^2-tests. The asymptotic results are based on large sample approximations of partial sums associated to sample covariances. For a bivariate time series such approximations are due to Kouritzin (1995). A high-dimensional linear process framework has been studied by Steland and von Sachs (2017) and Steland and von Sachs (2018). The results presented in this talk are based on large sample approximations which generalize those works by weakening assumptions and extending the results to a change-point model. All results hold for a linear process framework without constraints on the growths of the dimension of the vector time series and of the CUSUM transform. The asymptotic variances and covariances of the cumulated sums can be estimated consistently by a class of homogenous estimators. By studying consistency for sequential versions of these estimators, we are in a position to establish consistency of a stopped-sample estimator. Here the change-point estimator is used to segment the time series and the pre-change segment is used for estimation.

Finite sample properties are studied by simulations. As a real-world application and to illustrate the method, we analyze monitoring data from ozone sensors across the USA, where sensor data is compressed by projecting it onto sparse directions.

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Statistical Model Checks When Data Are Subject To Errors In Variables

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Abstract. In this paper we study statistical model checks when data are not fully observable. Particularly we shall consider a situation which in the mathematical literature is known as deconvolution while in statistics and econometrics is called errors in variables. For a brief introduction suppose that we have a sample $X_1, \ldots, X_n$ of i.i.d. replicates of a variable X with distribution function F. The nonparametric maximum likelihood estimator of F then equals the empirical distribution function

$$F_n(x) = \frac{1}{n} \sum_{i=1}^{n} 1_{\{X_i \leq x\}},$$

where 1_A is the indicator variable of the set A. In applied science it may happen that rather than the X_i we observe data contaminated by some error ε_i, say, i.e., we have

$$Y_i = X_i + \varepsilon_i, \qquad 1 \leq i \leq n.$$

Here, for each i, ε_i is independent of X_i with known distribution function H. Denoting with Y, X and ε representatives of the above variables, and letting G be the unknown distribution of Y, we obtain

$$G = F * H. \tag{1.11}$$

In other words, G is the convolution of F and H. It is then the goal to deconvolve equation (1.11) for analysis of F. Deconvolution is a classic example in an area which is often called "Inverse Problems". The approach is usually based on the fact that the associated characteristic functions Φ_X, Φ_Y and Φ_ε satisfy the relation

$$\Phi_Y = \Phi_X \Phi_\varepsilon$$

whence

$$\Phi_X = \frac{\Phi_Y}{\Phi_\varepsilon}. \tag{1.12}$$

Since along with H we also know Φ_ε and Φ_Y may be nonparametrically estimated from $Y_1, \ldots, Y_n$ we obtain nonparametric estimators for Φ_X. Most approaches to analyze errors in variables nonparametrically end up with inverting the estimator of (1.12). As it turns out, this is not possible without smoothing. Practically this approach has led us to an ill-posed inverse problem with all its bad consequences.

It is the purpose of this work to show that deconvolution is a well-posed problem so that rather than unacceptable rates of convergence or bad powers of tests the familiar $n^{-\frac{1}{2}}$

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rate may be achieved. In the talk we restrict ourselves to the question how to design model checks for F. Hence given a parametric model

$$\mathcal{M} = \{F_\theta : \theta \in \Theta\}$$

we discuss how to check the hypothesis

$$H_0 : F \in \mathcal{M}, \text{ i.e., } F = F_{\theta_0} \text{ for some } \theta_0.$$

This requires estimation of θ_0 and comparing a nonparametric quantity with a model depending analogue. In a sense this constitutes the analogue of Durbin (1973) to the convolution case. The estimation part is based on a general principle exploited in Stute (1986). Finally, we also study two sample situations and briefly comment on the regression situation.

Keywords: deconvolution, errors in variables, inverse problems

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On Tests for Multivariate Skewness and Kurtosis

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Abstract.

The third and fourth order cumulant vectors of a multivariate distribution, i.e. vectors composed, respectively, by all third and fourth order cumulants, contain all relevant information on multivariate skewness and kurtosis and it has been shown (Jammalamadaka et al. (2019)) that all the existing -cumulant based - indexes of skewness and kurtosis for multivariate distributions are based on these vectors.

In this talk we will concentrate on these cumulant vectors to give a unified treatment of the limit laws of different measures of multivariate skewness and kurtosis.

The analysis will exploit the Gram-Charlier Series to develop a Central Limit Theorem for the empirical multivariate skewness and kurtosis vectors. With this basic result it will be straightforward to obtain, via transformations, the limit distribution of several indexes of skewness and kurtosis.

The approach proposed here generalizes several existing results appeared in the literature and provide new ones; it will provide clear formulae for a large array of test statistics related to skewness and kurtosis of a multivariate distribution.

Symmetric distributions as well as asymmetric distributions will be considered: this will enable to derive asymptotic null distributions of tests as well as their power and efficiency by considering asymptotic distributions under alternatives.

Keywords: Gram-Charlier Series, Multivariate Skewness, Multivariate Kurtosis, Multivariate Ellyptically Symmetric Distributions, Multiple Cumulants, K-derivative

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Sequential monitoring for cointegrating regressions

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Abstract. In this paper we develop a family of monitoring procedures for a cointegrating regression, testing the null hypothesis of no breaks against the (non mutually exclusive) alternatives that there is either a change in the slope, or a change to a non-cointegrating relationship. Our procedure complements the monitoring scheme developed in Wagner and Wied (2017), who consider the same hypotheses. In particular, after observing the regression for a training sample m, we study a CUSUM-type test statistic, based on squares of residuals, to detect the presence of structural change during a monitoring horizon $m+1, ..., T$. Inspired by Lajos et al. (2004) and Lajos et al. (2007), our procedures are based on a class of boundary functions which depend on a parameter, $0 \leq \eta \leq \frac{1}{2}$, whose value affects the delay in detecting the possible break, with quicker detection as η approaches $\frac{1}{2}$. From a technical point of view, these procedures are based on almost sure limiting theorems (including a Darling-Erdős theorem), whose derivation is not straightforward. We therefore define a monitoring function which - at every point in time during the monitoring horizon - diverges almost surely to positive infinity under the null of no change, and drifts to zero almost surely under both alternatives. We then cast this sequence (as opposed to the original CUSUM process) into a randomising algorithm: the output is an *i.i.d.* sequence, with (asymptotically) finite moments of arbitrary order, for whose partial sums classical results can be shown (e.g. strong approximations). We then employ this newly generated sequence in order to define the detector function, showing that our monitoring procedure rejects the null of no break with a small probability if there is no break, whilst it rejects with probability one over the monitoring horizon in the presence of breaks. Monte Carlo evidence shows good size control and good power properties, with detection delays decidedly decreasing when using $\eta \geq 0$. We illustrate our procedure through an application to housing prices, showing that, in the early 2000s, fundamentals-driven cointegration relationships ceased to be valid, which suggests the emergence of a bubble in the US housing market.

Keywords: cointegration, structural change, sequential monitoring, randomized tests

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Various results on uniformity tests for directional data

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Abstract. Directional data are multivariate data for which only the directions (and not the magnitudes) are measured and which therefore belong to the unit sphere $\mathcal{S}^{p-1} := \{\mathbf{u} \in \mathbb{R}^p : \|\mathbf{u}\|^2 = \mathbf{u}'\mathbf{u} = 1\}$ of $\mathbb{R}^p$. In most practical applications the data lie on the circumference of the unit circle $\mathcal{S}^1$ (one then speaks of circular statistics) or on the surface of the unit sphere $\mathcal{S}^2$. Data of this type typically arise in meteorology (wind directions), astronomy (directions of cosmic rays or stars), earth sciences (paleomagnetism) and biology (protein structure, studies of animal navigation) to cite but these. A lot of attention in the literature has nevertheless been put on the general $\mathcal{S}^{p-1}$ case and even recently on high-dimensional hyperpheres. One of the most important problems in directional statistics is the problem of testing uniformity on $\mathcal{S}^{p-1}$. More precisely, the problem we are interested in here is to test $\mathcal{H}_0 : \mathrm{P} = \mathrm{Unif}(\mathcal{S}^{p-1})$ against $\mathcal{H}_1 : \mathrm{P} \neq \mathrm{Unif}(\mathcal{S}^{p-1})$, where P stand for the common probability distribution of an i.i.d. sequence $\mathbf{U}_1, \ldots, \mathbf{U}_n$ of unit random vectors and $\mathrm{Unif}(\mathcal{S}^{p-1})$ denotes the uniform probability distribution on $\mathcal{S}^{p-1}$. In view of its importance, there is a considerable literature on the topic. The most classical test for this problem is the Rayleigh test that rejects the null hypothesis for large values of $\|\bar{\mathbf{U}}\|$, where $\bar{\mathbf{U}} := n^{-1}\sum_{i=1}^n \mathbf{U}_i$. While the Rayleigh test is optimal against various types of alternatives, it is not *uniformly consistent* in the sense that it does not detect any departure of the null hypothesis of uniformity. The Rayleigh test belongs to a class of tests called the Sobolev tests. In this work we present various recent results related to some Sobolev tests. Both low and high-dimensional frameworks are considered.

Keywords: Directional statistics, Testing uniformity, Sobolev tests

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New weighted L^2-type tests for the inverse Gaussian distribution

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Abstract. We propose a new characterization of the inverse Gaussian distribution and associated families of goodness-of-fit tests. The proposed tests are weighted L^2-type tests depending on a tuning parameter. Asymptotic theory under the null, consistency for a broad class of alternatives and behaviour under contiguous alternatives is provided. Since the limit distributions depend on a parameter of the distribution, a parametric bootstrap procedure is proposed and a comparative simulation study shows that the new procedures are competitive to classical and recent tests.

Keywords: Goodness-of-fit, Inverse Gaussian distribution, Stein characterization

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Testing normality of data on a spatial grid

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Abstract. Despite the prevalence of the assumption of Gaussianity, there appears to exist no significance tests that could be used to assess if it is reasonable to assume that a given spatial data set can be treated as a realization of a Gaussian random field. This is a difficult problem because normality tests, and even exploratory tools like QQ-plots or histograms, require a random sample (iid observations) from a distribution whose Gaussianity is to be determined.

We propose a significance test to determine if data on a regular d–dimensional grid can be assumed to be a realization of Gaussian process. By accounting for the spatial dependence of the observations, we derive statistics analogous to sample skewness and kurtosis. We show that the sum of squares of these two statistics converges to a chi-square distribution with two degrees of freedom. This leads to a readily applicable test. We examine two variants of the test, which are specified by two ways the spatial dependence is estimated. We provide a careful theoretical analysis, which justifies the validity of the test for a broad class of stationary random fields.

We hope that the test we propose will turn out to be a useful diagnostic tool, which may lend confidence in the application of various methodologies based on the normality assumption, or provide a caution on the validity of conclusions. The test can be justified asymptotically using recent advances in the asymptotic theory for random fields and new arguments related to the quantification of spatial dependence. An appealing feature of our test is that the test statistics can be computed fairly easily using existing `R` software, and the critical values are those of a chi-square distribution.

A simulation study compares several implementations. While some implementations perform slightly better than others, all of them exhibit very good size control and high power, even in relatively small samples. An application to a comprehensive data set of sea surface temperatures further illustrates the usefulness of the test.

Keywords: Gaussian process, Significance test, Spatial grid

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Tests for conditional heteroscedasticity with functional data and goodness-of-fit tests for FGARCH models

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Abstract. Functional data objects that are derived from high-frequency financial data often exhibit volatility clustering characteristic of conditionally heteroscedastic time series. Versions of functional generalized autoregressive conditionally heteroscedastic (FGARCH) models have recently been proposed to describe such data, but so far basic diagnostic tests for these models are not available. We propose two portmanteau type tests to measure conditional heteroscedasticity in the squares of financial asset return curves. A complete asymptotic theory is provided for each test, and we further show how they can be applied to model residuals in order to evaluate the adequacy, and aid in order selection of FGARCH models. Simulation results show that both tests have good size and power to detect conditional heteroscedasticity and model mis-specification in finite samples. In an application, the proposed tests reveal that intra-day asset return curves exhibit conditional heteroscedasticity. Additionally, we found that this conditional heteroscedasticity cannot be explained by the magnitude of inter-daily returns alone, but that it can be adequately modeled by an FGARCH(1,1) model.

Keywords: Functional time series, Heteroscedasticity testing, Model diagnostic checking, High-frequency volatility models, Intra-day asset price

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MACE: Multiscale Abrupt Change Estimation under Complex Temporal Dynamics

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Abstract. We consider the problem of detecting abrupt changes in trend whereas the covariance and higher-order structures of the system can experience both smooth and abrupt changes in time. The number of jump points is allowed to diverge to infinity with the jump sizes possibly shrinking to zero. The method is based on a multiscale application of an optimal jump-pass filter to the time series, where the scales are dense between admissible lower and upper bounds. The MACE method is shown to be able to detect all possible jump points within a nearly optimal range with a prescribed probability asymptotically. For a time series of length n, the computational complexity of MACE is $O(n)$ for each scale and $O(n \log^{1.5} n)$ overall. Simulations and data analysis show that, under complex dynamics, MACE performs favorably compared with some of the state-of-the-art change point detection methods.

Keywords: change point estimation, multiscale detection, piece-wise locally stationary time series

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